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Photon polarization operator and quantum transitions in QCD vacuum

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Abstract. The analytical properties of the photon polarization operator in the quark vacuum was analyzed. Branching points of the polarization operator were found and its imaginary part was calculated. Quantum transitions between quark states and the turning over of a quark's color spin in a chromomagnetic field were predicted. The negative values of the photon energy and the contribution of the photon zero energy to the branching point were taken into account

Introduction

The investigation of the QCD vacuum is one of the current research areas in QCD. It is known that there are quark and gluon condensates in the QCD vacuum. These condensates are created with around them a color background; they play an important role in studies of the vacuum state. There are different models taking this background into account. The simplest and best investigated model is the vacuum in a constant chromomagnetic field. As we know, there are two ways of defining a non-abelian background: as a covariant constant field and as a field given by non-commuting vector potentials [1,12]. A great amount of works were devoted to the investigation of the vacuum properties, given the background field in terms of non-commuting potentials. It was found that the vacuum state in a constant chromomagnetic field, given by non-commuting potentials, is stable [9] and has a complicated topological structure [8]. These fields do not lead to spontaneously breaking of the chiral symmetry [11] etc.

A good tool for investigating the quark sector of the QCD vacuum is the photon polarization operator (PO) in a chromomagnetic field. From this some interesting information had been received. For example, in the SU(2) color group frame, it was observed that the PO in a spherical symmetric chromomagnetic field has an antisymmetric imaginary part [3]. This has been interpreted in [5] as a turning of the polarization plane of a photon in the chromomagnetic background (Faraday effect). The dispersion method [2] turned out to be very effective in exact calculating the imaginary part of the PO in the one-loop approximation [4,6]. Furthermore, this method was applied to calculations in the SU(3) color symmetry group [7]. But in all previous works [2,4,6,7] only the photon positive energy sector was studied, so not all effects taking place in the QCD vacuum with a chromomagnetic background have been revealed.

The aim of present work is to indicate the existence of one more vacuum phenomenon, namely the turning over of the color spin of vacuum quarks accompanying photon radiation or absorption, and the existence of new branching points of the PO corresponding to this phenomenon and to the photon's zero energy.

1 Green function of a quark in a chromomagnetic field

As noted before, a constant color background can be given by abelian or non-abelian vector potentials. In particular, non-abelian potentials giving a constant chromomagnetic field directed along the third axes of the ordinary and SU(3) color spaces has the form

$$A^\mu = A^\mu_a \frac{\lambda^a}{2}, \quad A^\mu_a = \tau^{1/2}\delta^\mu_a \quad a,\mu = 1,2,$$
$$A^0_a = A^3_a = 0, \quad a = 3,\cdots,8, \quad \tau = \text{const.} \qquad (1.1)$$

Here λ^a are the Gell-Mann matrices, Latin indices act in color space, and Greek ones are the Lorentz indices. For the potentials (1.1) only one component of the field strength tensor $F^a_{\mu\nu}$ differs from zero:

$$F^3_{12} = H^3_z = g\tau, \quad \text{and for the other} \quad F^a_{\mu\nu} = 0.$$

(g is the color interaction constant.)

In order to take into account the background (1.1) in the Green function of a quark we should have made the replacement $p_\mu \to P_\mu = p_\mu + gA_\mu$ in the expression

$$S(P) = \frac{1}{\gamma_\mu P^\mu - m}.$$

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After some simple transformations we get the form

$$
\begin{aligned}
S(P) &= (\gamma_\mu P^\mu + m)\left[\varLambda_1 - 2gp_\mu A^\mu - gF^3_{12}\Sigma^{12}\frac{\lambda^3}{2}\right] \\
&\quad\times \frac{I_2}{(p_0^2 - E_1^2)(p_0^2 - E_2^2)} + \frac{\gamma^\mu p_\mu + m}{p_0^2 - E_0^2} I_3 \\
&= S_{\mathrm{SU}(2)}(P) + S_{\mathrm{U}(1)}(P), \qquad (1.2)\\
E_0^2 &= \mathbf{p}^2 + m^2, \\
E_{1,2}^2 &= \mathbf{p}^2 + m^2 + \frac{g^2\tau}{2} \pm g\tau^{1/2}\sqrt{p_\perp^2 + \frac{g^2\tau}{2}}, \\
\Sigma^{12} &= \mathrm{i}\gamma^1\gamma^2, \\
\varLambda_1 &= p^2 - m^2 - \frac{g^2\tau}{2}, \\
I_2 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\
I_3 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\
p_\perp^2 &= p_1^2 + p_2^2;
\end{aligned}
$$

here the matrices I_2 and I_3 are defined in color space.

As we see from (1.2), the Green function of a quark in the background (1.1) is broken down into the SU(2) and U(1) terms that have been found earlier in [6]. The main advantage of expression (1.2) from the Green function obtained in [9] is that its pole part is separated from the matrix structure. This allows us to apply the dispersion method for the PO in the calculation [2].

From (1.2) it is clear too that the quark energy spectrum in the background (1.1) is split into three branches, E_0, E_1, E_2. The state E_0 is similar to the free quark state, E_1 and E_2 correspond to states where the projection of the quark's color spin $T^a = \lambda^a/2$ on the color vector

$$I^a = 2A^\mu_a p_\mu + gF^a_{\mu\nu}\Sigma^{\mu\nu} \qquad (1.3)$$

has a positive or negative sign, i.e. the sign of the last term in $E_{1,2}^2$ is defined by the scalar product $(T^a I^a)$. It should be noted that the state E_0 is absent in SU(2) theory and the vector I^a has the same form [6].

Threshold values of the energy for pair creation of a quark and antiquark from the same energy branch are
(a) $t_0 = (2E_0)^2_{\min} = 4m^2$,
(b) $t_1 = (2E_2)^2_{\min} = 4m^2$,
(c) $t_2 = (2E_1)^2_{\min} = 4(m^2 + g^2\tau)$;
for pair creation of a quark and antiquark from different branches of the energy spectrum
(d) $t_3 = (E_0 + E_2)^2_{\min} = 4m^2$,
(e) $t_4 = (E_0 + E_1)^2_{\min} = 2m^2 + g^2\tau + 2m(m^2 + g^2\tau)^{1/2}$,
(f) $t_5 = (E_1 + E_2)^2_{\min} = 2m^2 + g^2\tau + 2m(m^2 + g^2\tau)^{1/2}$;
and for the energy gap between these branches
(g) $t_6 = (E_1 - E_2)^2_{\min} = 2m^2 + g^2\tau - 2m(m^2 + g^2\tau)^{1/2}$
(h) $t_7 = (E_2 - E_0)^2_{\min} = 0$
(i) $t_8 = (E_1 - E_0)^2_{\min} = 2m^2 + g^2\tau - 2m(m^2 + g^2\tau)^{1/2}$.
(1.4) As we see from (1.4a), (1.4b) and (1.4d) at threshold energies t_0 pairs may be created from the branches E_0 and/or E_2, from (1.4e) and (1.4f) at energies t_4 from E_1 and E_0 (or E_2).

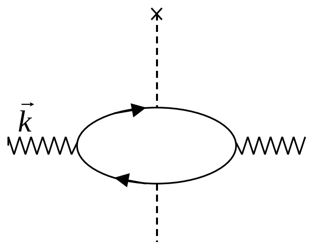

Fig. 1. A diagram of the photon PO in the one-loop approximation

2 Analytical properties of the photon polarization operator

A diagram of the photon PO in the one-loop approximation is drawn in Fig. 1. The internal lines are quark lines in the chromomagnetic field. According to Feynman's rules the PO can be written as in the following expression:

$$\varPi^\mu_\nu = -\mathrm{i}e^2 \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \mathrm{Sp}\left\{\gamma^\mu S(P)\gamma_\nu S(P-k)\right\}. \qquad (2.1)$$

We shall study the trace of the PO, $\varPi^\mu_\mu = \varPi(k^2,\tau)$, because its imaginary part is connected with physical processes. Substituting the Green function (1.2) into (2.1) we see that the photon PO in the background (1.1) is broken down into SU(2) and U(1) parts too, because of $I_2 I_3 = 0$, $I_2^2 = I_2$, $I_3^2 = I_3$. We have

$$\varPi^\mu_\nu[\mathrm{SU}(3)] = \varPi^\mu_\nu[\mathrm{SU}(2)] + \varPi^\mu_\nu[\mathrm{U}(1)]. \qquad (2.2)$$

After taking traces over the γ^μ and λ^a matrices we obtain

$$
\begin{aligned}
&\varPi^\mu_\mu(k^2) \qquad (2.3)\\
&= -\frac{\mathrm{i}e^2}{(2\pi)^4}\int \mathrm{d}^4 p \left\{\{16 f_1(p,k)\}/\left\{\left(p_0^2 - E_1^2\right)\right.\right. \\
&\times \left(p_0^2 - E_2^2\right)\left((p_0 - k_0)^2 - E_3^2\right)\left.\left((p_0 - k_0)^2 - E_4^2\right)\right\} \\
&+ \{8 f_2(p,k)\}/\left.\left\{\left(p_0^2 - E_0^2\right)\left((p_0 - k_0)^2 - E_5^2\right)\right\}\right\},
\end{aligned}
$$

$$
\begin{aligned}
f_1(p,k) &= \left(2m^2 - p(p-k)\right)\left(\varLambda_1\varLambda_2 + g^2\tau p_i (p-k)_i\right) \\
&+ \frac{g^2\tau}{2}\varLambda_1\left(p_i(p-k)_i + (p-k)_\perp^2 + \frac{g^2\tau}{2}\right) \\
&+ \frac{g^2\tau}{2}\varLambda_2\left(p_i(p-k)_i + p_\perp^2 + \frac{g^2\tau}{2}\right) + \frac{\left(g^2\tau\right)^2}{4} \\
&\times\left(p_\perp^2 + (p-k)_\perp^2 + 2p_i(p-k)_i + p(p-k)\right) \\
&+ \frac{g^2\tau}{2}\varLambda_1\varLambda_2 + \frac{\left(g^2\tau\right)^3}{8}, \\
f_2(p,k) &= 2m^2 - p(p-k), \quad i = 1,2,
\end{aligned}
$$

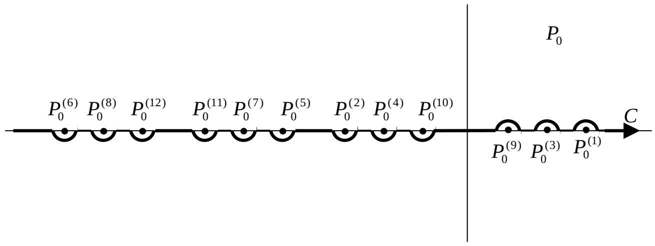

Fig. 2. Invoking the causality principle in the Green function

$$
\begin{aligned}
\Lambda_2 &= (p-k)^2 - m^2 - \frac{g^2\tau}{2}, \\
E_{3,4}^2 &= (\mathbf{p}-\mathbf{k})^2 + m^2 + \frac{g^2\tau}{2} \\
&\pm g\tau^{1/2}\sqrt{(p-k)_\perp^2 + \frac{g^2\tau}{4}}, \\
E_5^2 &= (\mathbf{p}-\mathbf{k})^2 + m^2, \quad p^2 = p_0^2 - \mathbf{p}^2.
\end{aligned}
$$

It is known that the photon PO in an external field is a function of the Lorentz invariants k^2, $k_\mu F^{\mu\nu}F_{\nu\alpha}k^\alpha$ and the field invariants [10]. However, all invariants of the field (1.1) are reduced to only different powers of the constant $g^2\tau$ and the dependence on them is not interesting for us. We shall study analyticity of the $\Pi(k^2) = \Pi_\mu^\mu$ function in the whole complex plane of $t = k^2$.

The function $\Pi(t)$ is single-valued in every point of the real t axis. In order to continue it onto complex values of t, it must single-valuedly be determined in every point of the complex t plane. At first let us consider the $t > 0$ area ($k_0^2 > \mathbf{k}^2$, a timelike photon). In this area we choose the reference frame $\mathbf{k} = 0$ and assume the field (1.1) given at this reference frame. (When passing to any other frame field (1.1) will acquire a chromoelectric component too.) In the frame $\mathbf{k} = 0$ the variable t is $t = k_0^2$ and does not depend on the sign of k_0. (Because of this the function $\Pi(t)$ was investigated only in the $k_0 > 0$ area in [2,4,6, 7].) As we will see later, the sign of the photon energy should be understood in terms of its absorption on $k_0 > 0$ and as its emission on $k_0 < 0$, and the area $k_0 < 0$ must be considered at the same instant as the area $k_0 > 0$. We shall consider $-\infty < k_0 < +\infty$ and assume $k_0 < -2E_1$.

Let us separate integration over p_0, as we see from (2.3) that all singularities of the function $\Pi(t)$ are situated in the p_0 plane:

$$
\begin{aligned}
\Pi\,(t) &= \int \mathrm{d}^3 p J \\
&= \int \mathrm{d}^3 p \left(-\frac{\mathrm{i}e^2}{(2\pi)^4}\right) \int\limits_c \mathrm{d}p_0 \left\{\{16 f_1(p,k_0)\}\right. \\
&\quad / \left\{ \left(p_0^2 - E_1^2\right)\left(p_0^2 - E_2^2\right)\left((p_0-k_0)^2 - E_1^2\right) \right. \\
&\quad \times \left. \left((p_0-k_0)^2 - E_2^2\right)\right\} + \{8 f_2(p,k_0)\} \\
&\quad / \left.\left\{\left(p_0^2 - E_0^2\right)\left((p_0-k_0)^2 - E_0^2\right)\right\}\right\}. \qquad (2.4)
\end{aligned}
$$

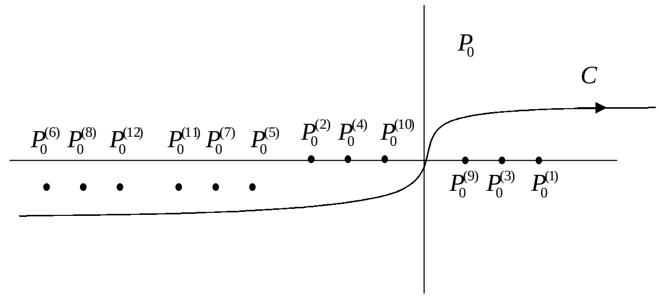

Fig. 3. Illustrating contour C on adding an infinitely small imaginary value iε to the variable t

From (2.4) it is seen that the integrand has poles at the following values of p_0:

$$
\begin{aligned}
p_0^{(1),(2)} &= \pm E_1, \quad p_0^{(3),(4)} = \pm E_2, \quad p_0^{(5),(6)} = k_0 \pm E_1, \\
p_0^{(7),(8)} &= k_0 \pm E_2, \quad p_0^{(9),(10)} = \pm E_0, \\
p_0^{(11),(12)} &= k_0 \pm E_0, \qquad (2.5)
\end{aligned}
$$

and the rule for path-tracing is given by the causality principle in the Green function (Fig. 2). It should be noted that the locations of the poles $p_0^{(7)}$ and $p_0^{(5)}$ principally differ from [6,7] because of the assumption $k_0 < -2E_1$. The poles $p_0^{(1)}, p_0^{(3)}$ and $p_0^{(9)}$ are located at a finite distance from the rest ones. When adding an infinitely small imaginary value iε to the variable t, the poles $p_0^{(5),(6)}, p_0^{(7),(8)}, p_0^{(11),(12)}$ will be displaced and by deforming the contour C they can avoid the intersection of contour C (Fig. 3). Let us observe the motion of these poles while k_0 is increasing. They all move along the p_0 axis from the left to the right.

(1) At $k_0 \to (E_0 - E_1) < 0$ the pole $p_0^{(5)}$ will approach the pole $p_0^{(9)}$ and the contour C will be squeezed by these poles (Fig. 4a). Avoiding intersection of contour C by the pole $p_0^{(5)}$ while adding iε to the variable t and consequently having a single-valued continuation of the function $\Pi(t)$ onto complex values of t becomes impossible. This circumstance could be eliminated by cutting the t plane along the real t axis since the point $t = (E_0 - E_1)^2_{\min} = t_8$ and the function $\Pi(t)$ will have two branches, $\Pi(t + \mathrm{i}0)$ and $\Pi(t - \mathrm{i}0)$. This means that the point $t = t_8$ is the branching point of the function $\Pi(t)$. Now the function $\Pi(t)$ is single-valued at every sheet of the t plane and it is possible to continue it to complex values of t analytically. But the contour C must be divided into two parts: $C = C^1 + C_0^{(1)}$ (Fig. 4b). However, on further increasing of k_0, more exactly

(2) at $k_0 \to (E_2 - E_1) < 0$, the pole $p_0^{(5)}$ together with $p_0^{(3)}$ will squeeze the contour $C_0^{(1)}$. One other branching point arises at $t = (E_2 - E_1)^2_{\min} = t_6$. Repeating the above analysis for a further increase of k_0 from the pole structure (see (2.5) and Fig. 2), more branching points of $\Pi(t)$ could be received because of squeezing of the next contours by the corresponding poles:

(3) $k_0 \to (E_0 - E_2) < 0, t = (E_0 - E_2)^2_{\min} = t_7$; the contour $C^1 = C^2 + C_0^{(2)}$, the poles $p_0^{(7)}$ and $p_0^{(9)}$;

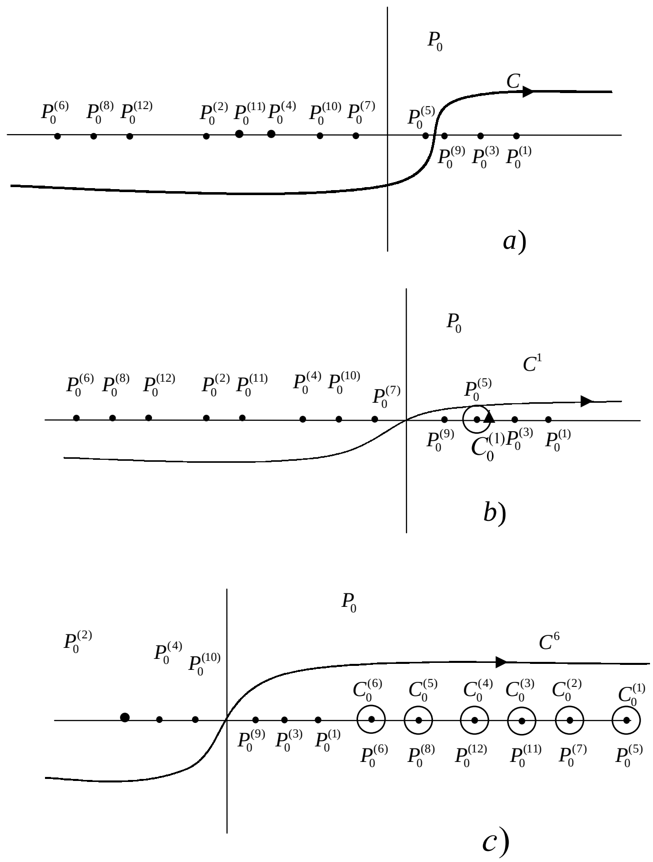

Fig. 4a–c. (a) Poles and contour with $k_0 \to (E_0 - E_1) < 0$; (b) division of contour in $C = C^1 + C_0^{(1)}$ and (c) in $C = C^6 + \sum_{i=1}^{6} C_0^{(i)}$

(4) $k_0 \to 0, t = 0 = t_9$; the contour $C^2 = C^3 + C_0^{(3)}$, the poles $p_0^{(11)}$ and $p_0^{(9)}$;

(5) $k_0 \to (E_2 - E_0) > 0,\ t = (E_2 - E_0)^2_{\min} = t_7$; the contour $C_0^{(3)}$, the poles $p_0^{(11)}$ and $p_0^{(4)}$;

(6) $k_0 \to (E_1 - E_2) > 0,\ t = (E_1 - E_2)^2_{\min} = t_6$; the contour $C_0^{(2)}$, the poles $p_0^{(7)}$ and $p_0^{(1)}$;

(7) $k_0 \to (E_1 - E_0) > 0, t = (E_1 - E_0)^2_{\min} = t_8$; the contour $C_0^{(3)}$, the poles $p_0^{(11)}$ and $p_0^{(1)}$;

(8) $k_0 \to 2E_0 > 0,\ t = (2E_0)^2_{\min} = t_0$; the contour $C^3 = C^4 + C_0^{(4)}$, the poles $p_0^{(12)}$ and $p_0^{(9)}$;

(9) $k_0 \to (E_0 + E_2) > 0,\ t = (E_0 + E_2)^2_{\min} = t_3$; the contour $C^4 = C^5 + C_0^{(5)}$, the poles $p_0^{(8)}$ and $p_0^{(10)}$;

(10) $k_0 \to 2E_2 > 0, t = (2E_2)^2_{\min} = t_1$; the contour $C_0^{(5)}$, the poles $p_0^{(8)}$ and $p_0^{(3)}$;

(11) $k_0 \to (E_0 + E_1) > 0,\ t = (E_0 + E_1)^2_{\min} = t_4$; the contour $C^5 = C^6 + C_0^{(6)}$, the poles $p_0^{(6)}$ and $p_0^{(9)}$;

(12) $k_0 \to (E_1 + E_2) > 0,\ t = (E_1 + E_2)^2_{\min} = t_5$; the contour $C_0^{(6)}$, the poles $p_0^{(6)}$ and $p_0^{(3)}$;

(13) $k_0 \to (2E_1) > 0,\ t = (2E_1)^2_{\min} = t_2$; the contour $C_0^{(6)}$, the poles $p_0^{(6)}$ and $p_0^{(1)}$. In the end the contour C is divided into seven parts $C = C^6 + \sum_{i=1}^{6} C_0^{(i)}$ (Fig. 4c).

So, taking into account $k_0 < 0$, we get ten branching points instead of three [6,7]. If we assume $k_0 > 0$ at the beginning of our analysis and observe the motion of the poles in the opposite direction, i.e. with $k_0 \to -\infty$, we see that the branching points (8)–(13) arise on negative values of k_0 as well,

(14) $k_0 = -2E_0$,
(15) $k_0 = -(E_0 + E_2)$,
(16) $k_0 = -(E_0 + E_1)$,
(17) $k_0 = -2E_2$,
(18) $k_0 = -(E_1 + E_2)$,
(19) $k_0 = -2E_1$,

because of squeezing of the contour. All photon energies (1)–(19) have a physical meaning and correspond to a physical phenomenon.

For the whole investigation of the analytical continuation of the function $\Pi(t)$ we should consider negative values of t too. For the area $t < 0$ ($k_0^2 < \mathbf{k}^2$, a space-like photon) we may choose a reference frame in which $k_0 = 0, t = -\mathbf{k}^2$. Then the integrand in (2.3) has the following poles:

$$
\begin{aligned}
p_0^{(1),(2)} &= \pm E_1, \quad p_0^{(3),(4)} = \pm E_2, \quad p_0^{(5),(6)} = \pm E_3, \\
p_0^{(7),(8)} &= \pm E_4, \quad p_0^{(9),(10)} = \pm E_0, \\
p_0^{(11),(12)} &= \pm E_5.
\end{aligned}
$$

While adding an infinitely small imaginary value $\mathrm{i}\varepsilon$ to the variable k^2, the poles $p_0^{(5)}$–$p_0^{(8)}$ and $p_0^{(11),(12)}$ will be displaced and approach the contour C. However, even numbered poles will remain at a finite distance from the odd numbered ones at every value of $k^2 \Big(d = p_0^{(i)} - p_0^{(j)} = E_i - (-E_j) = E_i + E_j > 0\Big)$.

By deforming of the contour C we could avoid its intersection by the poles, and there is no contour squeezing by the poles. Consequently, there are no branching points of $\Pi(t)$ at the $t < 0$ area and the function $\Pi(t)$ may be single-valuedly continued to the complex half-plane $t < 0$.

3 The imaginary part of the polarization operator

Let us calculate the imaginary part of the PO using the above analysis. Namely $\mathrm{Im}\Pi(t)$ is connected with the full cross section of the e^+e^- annihilation into hadrons in the condensate background, with virtual photon decay to a quark–antiquark pair. Besides, knowing $\mathrm{Im}\Pi(t)$ due to the dispersion relation (see [2,6,7])

$$
\Pi(t) = \frac{t^2}{\pi} \int \mathrm{d}x \frac{\mathrm{Im}\Pi(x)}{x^2(x^2 - t)},
$$

the real part of $\Pi(t)$ could be found as well.

From (2.4) we have seen that at real values of t the function $\Pi(t)$ is real, i.e. $\Pi(t^*) = \Pi^*(t)$. Near the cut-line this will be

$$\Pi(t - \mathrm{i}0) = \Pi^*(t + \mathrm{i}0) \tag{3.1}$$

Using (3.1) the relation between the jump of the $\Pi(t)$ function when passing the cut-line and its imaginary part could be found:

$$\begin{aligned}\Delta\Pi(t) &= \Pi(t+\mathrm{i}0) - \Pi(t-\mathrm{i}0)\\ &= \Pi(t+\mathrm{i}0) - \Pi^*(t+\mathrm{i}0)\\ &= 2\mathrm{i}\mathrm{Im}\Pi(t).\end{aligned} \tag{3.2}$$

From the jump definition (3.2) it can be seen that only the integrals over the contours $C_0^{(i)}$ will contribute to the jump $\Delta\Pi(t)$, because integrals over the C^i contours will mutually cancel:

$$\begin{aligned}J_0 = &-\frac{\mathrm{i}e^2}{(2\pi)^4}\int\limits_{\sum_i c_0^{(i)}} \mathrm{d}p_0 \left\{\{8f_2(p,k_0)\}/\left\{(p_0^2 - E_0^2)\right.\right.\\ &\times \left.\left((p_0-k_0)^2 - E_0^2\right)\right\} + \{16 f_1(p,k_0)\}\\ &/ \left\{(p_0^2 - E_1^2)(p_0^2 - E_2^2)\left((p_0-k_0)^2 - E_1^2\right)\right.\\ &\times \left.\left.\left((p_0-k_0)^2 - E_2^2\right)\right\}\right\}.\end{aligned} \tag{3.3}$$

The integral J_0 is easily calculated by means of the residue theorem

$$\begin{aligned}J_0 = &(2\pi\mathrm{i})\sum_{i=5,6,7,8,11,12}\mathrm{res}\left(p_0^{(i)}\right)\\ = &-\frac{e^2}{2\pi^3 k_0}\left\{\frac{1}{E_1(k_0-2E_1)}\left[m^2 + k_0E_1 + \frac{(g^2\tau)^{3/2}}{2}\right.\right.\\ &\times\left.\frac{p_\perp^2 + \frac{g^2\tau}{4} + g\tau^{1/2}\sqrt{p_\perp^2 + \frac{g^2\tau}{4}} - k_0E_1 + \frac{g^2\tau}{4}}{\sqrt{p_\perp^2 + \frac{g^2\tau}{4}}(k_0 - (E_1+E_2))(k_0-(E_1-E_2))}\right]\\ &+ \frac{1}{E_2(k_0-2E_2)}\left[m^2 + k_0E_2 - \frac{(g^2\tau)^{3/2}}{2}\right.\\ &\times\left.\frac{p_\perp^2 + \frac{g^2\tau}{4} - g\tau^{1/2}\sqrt{p_\perp^2 + \frac{g^2\tau}{4}} - k_0E_2 + \frac{g^2\tau}{4}}{\sqrt{p_\perp^2 + \frac{g^2\tau}{4}}(k_0 - (E_1+E_2))(k_0-(E_2-E_1))}\right]\\ &+\left.\frac{2}{E_0(k_0-2E_0)}\left[m^2 + k_0E_0\right]\right\}\\ &+ (k_0 \to -k_0).\end{aligned} \tag{3.4}$$

Using in (3.4) Sokhotski's formula

$$\frac{1}{x+\mathrm{i}0} = P\frac{1}{x} - \mathrm{i}\pi\delta(x)$$

in the branching points $k_0 = \pm(2E_0)$, $\pm(2E_2)$, $\pm(2E_1)$, $\pm(E_1 \pm E_2)$ we find ΔJ_0, the jump value of J_0:

$$\begin{aligned}\Delta J_0 = &\frac{-e^2}{\pi k_0}\left\{\frac{\delta(k_0-2E_1)}{E_1}\left[m^2 + k_0E_1 + \frac{(g^2\tau)^{3/2}}{2}\right.\right.\\ &\times\left.\frac{p_\perp^2 + \frac{g^2\tau}{4} + g\tau^{1/2}\sqrt{p_\perp^2 + \frac{g^2\tau}{4}} - k_0E_1 + \frac{g^2\tau}{4}}{\sqrt{p_\perp^2 + \frac{g^2\tau}{4}}(k_0 - (E_1-E_2))(k_0-(E_1+E_2))}\right]\\ &+ \frac{\delta(k_0-2E_2)}{E_2}\left[m^2 + k_0E_2 - \frac{(g^2\tau)^{3/2}}{2}\right.\\ &\times\left.\frac{p_\perp^2 + \frac{g^2\tau}{4} - g\tau^{1/2}\sqrt{p_\perp^2 + \frac{g^2\tau}{4}} - k_0E_2 + \frac{g^2\tau}{4}}{\sqrt{p_\perp^2 + \frac{g^2\tau}{4}}(k_0 - (E_2-E_1))(k_0-(E_1+E_2))}\right]\\ &+ \frac{2\delta(k_0-2E_0)}{E_0}\left[m^2 + k_0E_0\right] + \frac{(g^2\tau)^{3/2}}{2}\\ &\times\left[\frac{p_\perp^2 + \frac{g^2\tau}{4} + g\tau^{1/2}\sqrt{p_\perp^2 + \frac{g^2\tau}{4}} - k_0E_1 + \frac{g^2\tau}{4}}{E_1\sqrt{p_\perp^2 + \frac{g^2\tau}{4}}(k_0 - (E_1-E_2))(k_0-2E_1)}\right.\\ &-\left.\frac{p_\perp^2 + \frac{g^2\tau}{4} - g\tau^{1/2}\sqrt{p_\perp^2 + \frac{g^2\tau}{4}} - k_0E_2 + \frac{g^2\tau}{4}}{E_2\sqrt{p_\perp^2 + \frac{g^2\tau}{4}}(k_0 - (E_2-E_1))(k_0-2E_2)}\right]\\ &\times\delta(k_0 - (E_1+E_2)) + \frac{(g^2\tau)^{3/2}}{2}\\ &\times\frac{p_\perp^2 + \frac{g^2\tau}{4} + g\tau^{1/2}\sqrt{p_\perp^2 + \frac{g^2\tau}{4}} - k_0E_1 + \frac{g^2\tau}{4}}{E_1\sqrt{p_\perp^2 + \frac{g^2\tau}{4}}(k_0 - (E_1+E_2))(k_0-2E_1)}\\ &\times\left.\delta(k_0-(E_1-E_2))\right\}\Theta(k_0)\\ &+ (k_0 \to -k_0)\Theta(-k_0).\end{aligned} \tag{3.5}$$

+Having integrated the expression (3.5) over $\mathbf{p}$ we find the jump $\Delta\Pi(t)$ and then by means of (3.2) the imaginary part of the $\Pi(t)$ function:

$$\begin{aligned}&\mathrm{Im}\Pi(t)\\ &= \frac{-e^2}{2\pi t^{1/2}}\left\{\left[\frac{1}{2}\left(t + \frac{t_1}{2} + \frac{g^2\tau}{2}\right)\sqrt{t-t_2} - \frac{g\tau^{1/2}}{2}\right.\right.\\ &\times\left(t + \frac{t_1}{2} - \frac{g\tau^{1/2}}{2}\right)\left(\frac{\pi}{2} - \arcsin\frac{2g\tau^{1/2}}{\sqrt{t-t_1}}\right) + \frac{g^2\tau}{2}\\ &\times\frac{t - \frac{g^2\tau}{2}}{\sqrt{t - t_1 - g^2\tau}}\\ &\times\left.\ln\frac{g\tau^{1/2}\sqrt{t-t_1}}{t - t_1 - 2g^2\tau + \sqrt{(t-t_1-g^2\tau)(t-t_2)}}\right]\\ &\times\Theta(t-t_2) + \left[\frac{1}{2}\left(t + t_1 + \frac{g^2\tau}{2}\right)\sqrt{t-t_1} + g\tau^{1/2}\frac{\pi}{4}\right.\\ &\times\left.\left(t + \frac{t_1}{2} - \frac{g^2\tau}{2}\right) - \frac{g^2\tau}{2}\left(t - \frac{g^2\tau}{2}\right)A(t)\right]\Theta(t-t_1)\\ &-\left[\frac{g^2\tau}{2}\frac{t}{t-g^2\tau}\sqrt{t - t_1 - g^2\tau\left(2 - \frac{g^2\tau}{t}\right)}\right.\end{aligned}$$

$$
\begin{aligned}
&- \frac{g^2\tau\left(t-\dfrac{g^2\tau}{2}\right)}{\sqrt{t-t_1-g^2\tau}} \\
&\times \ln \frac{\sqrt{t-t_1-g^2\tau}-\sqrt{t-t_1-g^2\tau\left(2-\dfrac{g^2\tau}{t}\right)}}{g\tau^{1/2}\sqrt{1-\dfrac{g^2\tau}{t}}}\Bigg] \\
&\times \Theta(t-t_5) - \frac{g^2\tau}{2}\left(1-\frac{g^2\tau}{t}\right)^2 \\
&\times \frac{\sqrt{t^2-g^4\tau^2}}{\sqrt{2g^2\tau t - g^2\tau t_1 - t^2 - g^4\tau^2}}\Theta(t-t_6)\Bigg\}, \qquad (3.6)
\end{aligned}
$$

$$
A(t) = \begin{cases} \dfrac{1}{\sqrt{t-t_1-g^2\tau}} \ln \dfrac{\sqrt{t-t_1}+\sqrt{t-t_1-g^2\tau}}{g\tau^{1/2}}, \\ \qquad t > t_1 + g^2\tau, \\ \dfrac{1}{\sqrt{t_1+g^2\tau-t}}\left(\dfrac{\pi}{2} - \arcsin\dfrac{\sqrt{t-t_1}}{g\tau^{1/2}}\right), \\ \qquad t < t_1 + g^2\tau. \end{cases}
$$

This formula is correct for all values of k_0. If we would consider only positive values of k_0, we would have got the same expression for $\mathrm{Im}\Pi(t)$. It should be noted that $\mathrm{Im}\Pi(t)$ turns out to be finite, because the virtual quarks are the main reason that the divergences did not contribute to its expression due to the δ functions.

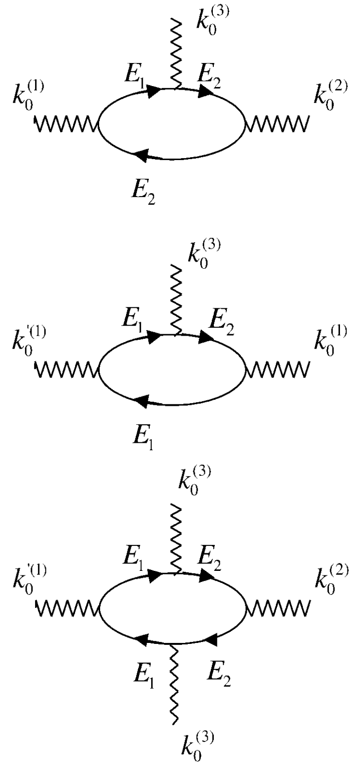

Fig. 5. Graphically illustrating photon splitting into two or three photons in the condensate background

4 Turning over of quark color spin and transitions

As we see from the list of branching points, the function $\Pi(t)$ has a new type of branching points, $t_k = (E_i - E_j)^2_{\min}$, which arose due to taking into account the $k_0 < 0$ area; these are absent in our previous works [4,6,7]. The branching point $t_6 = (E_1 - E_2)^2_{\min}$ corresponds to the photon energy necessary for the quark transitions from state E_2 to state E_1 ($k_0 = E_1 - E_2$) and vice versa ($k_0 = E_2 - E_1$). Since E_1 and E_2 correspond to quark energies, when the color spin of a quark is directed along (the E_1 state) or opposite (the E_2 state) the color vector (1.3), these transitions mean the turning over of the color spin of the quark when absorbing ($k_0 = E_1 - E_2 > 0$) or emitting ($k_0 = E_2 - E_1 < 0$) the photon. As we see, the sign of the photon energy k_0 is connected to its absorption and emission. So, a photon not having enough energy for pair creation may be absorbed by a vacuum quark having changed its color spin direction (transition $E_2 \to E_1$). Then the quark having returned to its lower energy state (transition $E_1 \to E_2$) emits this photon. The photon "feels" a background due to the splitting energy spectrum of the vacuum quarks. The probability of such transitions may be defined by the last addendum of (3.6) by means of the formula

$$W = -\frac{1}{k_0}\mathrm{Im}\Pi(k^2), \qquad (4.1)$$

and analogously for pair creation.

While analyzing $\delta(k_0 - (E_1 - E_2))$ we see that there are connections between the transverse momenta of the quark $p_\perp$, the photon energy k_0 and the field strength $g\tau^{1/2}$,

$$p_\perp^2 = \frac{1}{4}\left(\frac{k_0^4}{g^2\tau} - g^2\tau\right), \qquad (4.2)$$

i.e. at fixed values of the photon energy and field strength, only quarks whose transverse momentum obeys (4.2) may make the transition $E_1 \leftrightarrow E_2$. In particular, when $k_0^2 = g^2\tau$ only quarks resting on the (x,y) plane ($p_\perp = 0$) make this transition and the origin of the singularity arising in (3.6) at $t = g^2\tau$ is connected with this circumstance.

The transition $E_1 \leftrightarrow E_2$ predicts another interesting phenomenon connected with the QCD vacuum and may be observed in experiment or in nature. This is photon splitting into two or three photons in the condensate background:

$$
\begin{aligned}
k_0^{(1)} &= E_1 + E_2 = 2E_2 + (E_1 - E_2) = k_0^{(2)} + k_0^{(3)}, \\
k_0^{\prime(1)} &= 2E_1 = (E_1 + E_2) + (E_1 - E_2) = k_0^{(1)} + k_0^{(3)}, \\
k_0^{\prime(1)} &= 2E_1 = 2E_2 + 2\,(E_1 - E_2) = k_0^{(2)} + 2k_0^{(3)}.
\end{aligned}
$$

Graphically it may be drawn as in Fig. 5.

There are vacuum state reconstruction effects in QED [13]; while switching on a magnetic field the vacuum electrons transit to a lower energy level. The analog of this effect may be also exist in the QCD vacuum due to $E_1 \to E_2$ transitions with photon emitting.

The branching points $k_0 = \pm 2E_{0,1,2}$ correspond to pair creation (annihilation) when a quark and antiquark from the same energy branch and $k_0 = \pm(E_0 + E_{1,2})$ when they are from different ones. The branching points $k_0 = \pm(E_1 - E_0)$ and $k_0 = \pm(E_2 - E_0)$ correspond to the transitions $E_0 \leftrightarrow E_1$ and $E_0 \leftrightarrow E_2$. They have no physical meaning for turning over of the color spin of the quark. It should be noted that the poles at values $k_0 = \pm(E_0 \pm E_{1,2})$ in expression (3.4) of J_0 did not arise, and consequently there are no addenda in the expression of $\mathrm{Im}\Pi(t)$, see (3.6), corresponding to these branching points. This is connected with (2.2), i.e. a full separation of SU(2) and U(1) terms. This means that there are no transitions $E_0 \leftrightarrow E_{1,2}$ by absorbing or emitting a photon, and no creation or annihilation of a quark and antiquark from the energy branches E_0 and $E_{1,2}$. This is a result of the accuracy of the color symmetry, and the quark's color cannot be changed by photon absorbing or emitting. If we consider pure QCD, i.e. replace the electromagnetic vertices $e\gamma^\mu$ with chromodynamic ones $g\gamma^\mu\lambda^a/2$ in (2.1), we get the PO of a gluon (only the quark loop part). The equality (2.2) will not hold for the gluon PO, and transitions, creations and annihilations $k_0 = \pm(E_{1,2} \pm E_0)$ will take place by gluon emitting or absorbing. The vacuum state reconstruction effects in QCD may occur due to $E_{1,2} \to E_0$ transitions as well.

5 Contribution of virtual quarks

The other new branching point is $k_0 = 0$. The $k_0 = 0$ point arises in pure QED on investigating the electron–positron vacuum as an ordinary singularity, not as a branching point [2] because we do not consider the $k_0 < 0$ area. As we put $\mathbf{k} = 0$ this means the photon is absent and quarks are created and annihilated and make transitions spontaneously, in the photon's absence. Having used Sokhotski's formula in (3.4) we find that the jump of J_0 corresponding to this branching point is

$$\begin{aligned}\Delta J_0 = &-\frac{e^2\mathrm{i}}{\pi^2}\left\{\frac{1}{E_1^2}\right.\\ &\times\left[m^2+\frac{g^2\tau}{4}\left(1+\frac{g\tau^{1/2}}{\sqrt{p_\perp^2+\frac{g^2\tau}{4}}}+\frac{g^2\tau}{4}\frac{1}{p_\perp^2+\frac{g^2\tau}{4}}\right)\right]\\ &+\frac{1}{E_2^2}\left[m^2+\frac{g^2\tau}{4}\left(1-\frac{g\tau^{1/2}}{\sqrt{p_\perp^2+\frac{g^2\tau}{4}}}+\frac{g^2\tau}{4}\frac{1}{p_\perp^2+\frac{g^2\tau}{4}}\right)\right]\\ &\left.+\frac{2m^2}{E_0^2}\right\}\delta(k_0). \qquad (5.1)\end{aligned}$$

As can be seen from (5.1) the δ function corresponding to this pole is $\delta(k_0)$ and is not connected with k_0 and $\mathbf{p}$. Of course, this leads to a divergence of $\Delta\Pi(0)$ corresponding to this point. Having integrated (5.1) over $\mathbf{p}$ and using (3.2) we find $\mathrm{Im}\Pi(0)$:

$$\begin{aligned}\mathrm{Im}\Pi(0) = &-\frac{e^2}{2\pi^2}\left(m^2+\frac{g^2\tau}{4}\right)\int \mathrm{d}^3p\left(\frac{1}{E_1^2}+\frac{1}{E_2^2}\right)\\ &-\frac{e^2}{\pi^2}m^2\int \mathrm{d}^3p\frac{1}{E_0^2}+\frac{e^2}{2\pi}(g\tau^{1/2})\mathrm{Arsh}\frac{2g\tau^{1/2}}{m}\\ &-\frac{e^2}{2\pi}\left(\frac{g^2\tau}{4}\right)^2\frac{1}{\sqrt{m^2+\frac{g^2\tau}{4}}}\\ &\times\left[\mathrm{Arsh}\frac{2m}{g\tau^{1/2}}+\mathrm{Arsh}\frac{2m^2+g^2\tau}{g\tau^{1/2}m}\right]. \qquad (5.2)\end{aligned}$$

Since only virtual quarks lead to a divergence, the expression (5.2) may be considered as the contribution of virtual quarks to the imaginary part of the PO and by means of (4.1) it describes the probability of spontaneous pair creation and annihilation from the vacuum and spontaneous transitions in the QCD vacuum.

The $k_0 < 0$ area should be considered, and the branching point $k_0 = 0$ should be taken into account in pure QED for the investigation of the electron–positron vacuum as well.

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